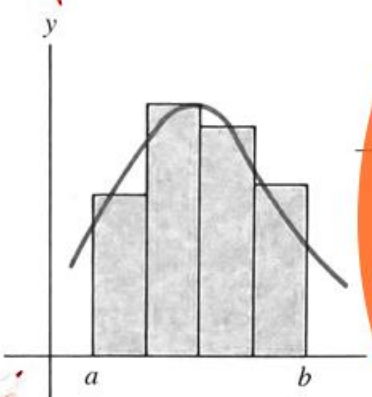
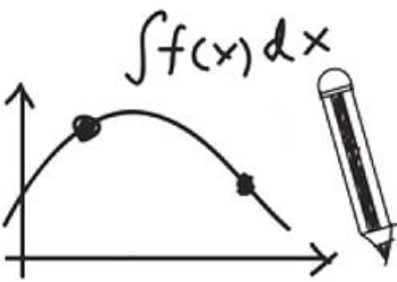


Calculus(I)

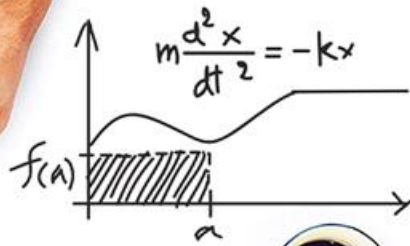
$$x^2 - 3x - 4 = 0$$

$$4x^2 - 3x - 1 = 0$$



$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

$$F = mg = ma = m \frac{d^2h}{dt^2}$$



Gottfried Wilhelm Leibniz

$$\frac{dA}{dt} = \frac{dB}{dt} = -\frac{dC}{dt} = \frac{dD}{dt} = (c_1)T^{\frac{1}{2}}AB - (c_2)T^{\frac{1}{2}}CD$$

$$m \frac{d^2x}{dt^2} = -kx - f \frac{dx}{dt} + A \sin(\omega t)$$

$$y' = \text{and } v' = -ky - fv + A \sin(\omega t)$$

$$m \frac{d^2x}{dt^2} = -kx$$

$$x = A \frac{dT}{dt} - (c_1)(T - T)$$

$$\frac{df(x)}{dx}$$

$$\frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$Lx + h, f(x) + 1$$



Some Trigonometric Integrals

Lecturer: Xue Deng

How to compute these integrations?

$$\int \sin^n x dx = ?$$

$$\int \cos mx \cos nx dx = ?$$

$$\int \cos^n x dx = ?$$

$$\int \sin mx \sin nx dx = ?$$

$$\int \sin mx \cos nx dx = ?$$

$$\int \sin^m x \cos^n x dx = ?$$

Basic Identities

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Half-Angle Identities

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

Basic Identities

Product Identities


$$(1) \sin mx \cos nx = \frac{1}{2} [\sin(m+n)x + \sin(m-n)x]$$

$$(2) \sin mx \sin nx = -\frac{1}{2} [\cos(m+n)x - \cos(m-n)x]$$

$$(3) \cos mx \cos nx = \frac{1}{2} [\cos(m+n)x + \cos(m-n)x]$$

Example 1

Find $\int \sin^5 x dx$ (n odd)

 $= \int \sin^4 x \sin x dx = -\int \sin^4 x d \cos x$

$$= -\int (1 - \cos^2 x)^2 d \cos x$$


Polynomial of $\cos x$

$$= -\int (1 - 2\cos^2 x + \cos^4 x) d \cos x$$

$$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$$

Example 2

Find $\int \sin^2 x dx$ (n even)

 $= \int \frac{1 - \cos 2x}{2} dx$


Half-Angle Identity

$$= \frac{1}{2} \int (1 - \cos 2x) dx$$

$$= \frac{1}{2} x - \frac{1}{4} \sin 2x + C$$

Example 3

Find $\int \sin^2 x \cos^4 x dx$ (Both m and n even)


 $= \int \left(\frac{1 - \cos 2x}{2} \right) \left(\frac{1 + \cos 2x}{2} \right)^2 dx$ **Half-Angle Identities**

$$= \frac{1}{8} \int \left[\frac{1}{2} - \frac{1}{2} \cos 4x + \sin^2 2x \cos 2x \right] dx$$

$$= \frac{1}{8} \left(\frac{1}{2} x - \frac{1}{8} \sin 4x + \frac{1}{6} \sin^3 2x \right) + C$$

Example 4

Find $\int \sin 2x \cos 3x dx$

 $= \frac{1}{2} \int \sin(5x) + \sin(-x) dx$

Product Identity

$$= \frac{1}{10} \int \sin 5x d5x - \frac{1}{2} \int \sin x dx$$

$$= -\frac{1}{10} \cos 5x + \frac{1}{2} \cos x + C$$

Summary

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Half-Angle Identities

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

Summary

Product Identities

$$(1) \sin mx \cos nx = \frac{1}{2} [\sin(m+n)x + \sin(m-n)x]$$

$$(2) \sin mx \sin nx = -\frac{1}{2} [\cos(m+n)x - \cos(m-n)x]$$

$$(3) \cos mx \cos nx = \frac{1}{2} [\cos(m+n)x + \cos(m-n)x]$$

Summary

$$\int \sin^m x \cos^n x dx$$


When m, n has an odd number, look for a substitution;

when m, n are both even number, use half-angle identities.

Questions and Answers

Q1: $\int \sin x \cos x dx$

Method 1

 $= -\int \cos x (-\sin x) dx$

The diagram shows a pink box around the term $(-\sin x)$ in the integral. A pink curved arrow points from the top of this box to the dx term, indicating the substitution $u = -\sin x$.

substitution

$$= -\int \cos x d \cos x$$

$$= -\frac{1}{2} \cos^2 x + C$$

Questions and Answers

Q1: $\int \sin x \cos x dx$

Method2



$$= \int \sin x (\cos x) dx$$




substitution

$$= \int \sin x d \sin x$$

$$= \frac{1}{2} \sin^2 x + C$$

Questions and Answers

Q2: $\int_{-\pi}^{\pi} \sin mx \sin nx dx$ (m and n are positive integers)


 $= -\frac{1}{2} \int_{-\pi}^{\pi} [\cos(m+n)x - \cos(m-n)x] dx$ **Case 1: $m \neq n$**

$$= -\frac{1}{2} \left[\frac{1}{m+n} \sin(m+n)x - \frac{1}{m-n} \sin(m-n)x \right]_{-\pi}^{\pi}$$

$$= 0$$

Questions and Answers

Q2: $\int_{-\pi}^{\pi} \sin mx \sin nx dx$ (m and n are positive integers)

 $= -\frac{1}{2} \int_{-\pi}^{\pi} [\cos 2mx - 1] dx$

Case 2: $m = n$

$$= -\frac{1}{2} \left[\frac{1}{2m} \sin 2mx - x \right]_{-\pi}^{\pi}$$

$$= \pi$$

Some Trigonometric Integrals

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